IV. *Experiments on the Value of the British Association Unit of Resistance.*

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Part I.

Before leaving Cambridge, Professor Chrystal, in the summer of 1878, wound with great care two large coils of about 50 centims. in diameter to be used for a redetermination of the value of the British Association unit of resistance. A galvanometer of special construction with double coils, one of thick wire the other of thin, was also wound by him and mounted with Weber's suspension.

The coils were to be placed at a known distance apart, so that the coefficient of mutual induction could be calculated.

The apparatus remained unused till the spring of last year, when the experiments described in the present paper were commenced.

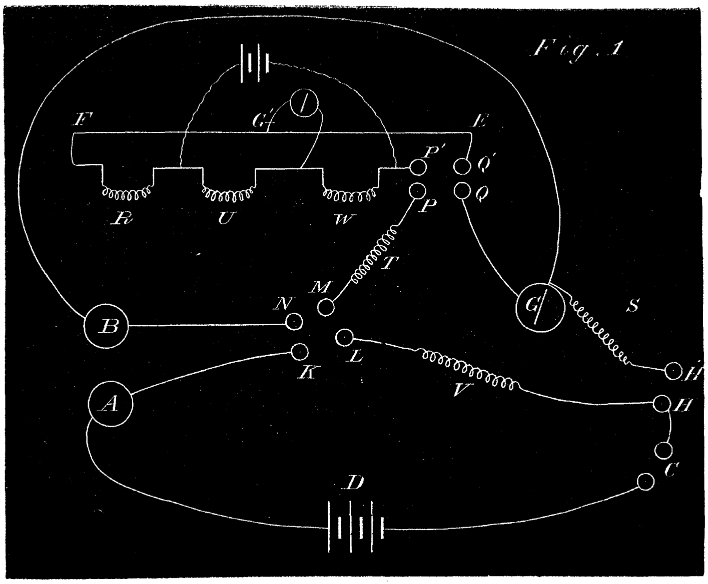
The method used is similar to those employed by Kirchhoff (Pogg. Ann., lxxvi.) and Rowland (American Journal of Science and Arts, vol. xv., 1878). The coefficient of mutual induction of the two coils is determined by calculation from the geometrical data. A current is passed through one coil, the other being in circuit with a ballistic galvanometer, and the induced current, produced when the primary current is broken or reversed, is measured by the throw of the galvanometer needle. The primary current itself being then measured by some method, we have enough data to determine in absolute measure the resistance of the secondary circuit and the galvanometer. But this resistance can be measured in terms of the B.A. unit, and hence a value obtained for the latter.

Professor Rowland measured his primary current by means of a second galvanometer, the constant of which he compared with that of the ballistic galvanometer both by direct experiment and by calculation.

In our experiments the value of the primary current was obtained by passing by means of a shunt about $\frac{1}{3000}$th part of it through the ballistic galvanometer; we thus eliminate from our equations the values of the galvanometer constants, as well as the correction which ROWLAND had to apply for the difference in the intensity of the horizontal component of the earth's magnetic force at the two galvanometers.

We have instead to determine exactly the ratio between two resistances of about 1 and 3000 B.A. units respectively, and to show that the heating of the wires by the current could never be such as to affect the value of this ratio appreciably. We proceed to describe the arrangement of the apparatus.

Fig. 1 gives a diagrammatic plan. A and B are the two coils, A being the primary. G is the galvanometer. K, L, M, N are four mercury cups.



P, Q, H, and H′ are also mercury cups. C is the commutator, and D the battery. Between H′ and G is a resistance, S, of about 3000 B.A. units, between H and L a second resistance, V, of about 1 unit.

Between P and M there is an adjustable resistance, T, the purpose of which will be described shortly.

The rest of the figure represents the ordinary WHEATSTONE'S bridge arrangement for measuring the resistance of the secondary circuit. P′, Q′ are two mercury cups. E, F a divided wire. R a resistance of about 160 B.A. units (the total resistance of the secondary circuit). U and W two equal resistances of about 30 units, forming the other arms of the bridge.

Let us suppose P P′, Q Q′, and M N are connected by stout copper ┌┴┐-shaped pieces. Then our secondary circuit, broken between P and Q, forms the fourth arm of the WHEATSTONE'S bridge, and by adjusting the variable resistance, T, the resistance of the secondary circuit can be made so nearly equal to R that the difference between them may be expressed in terms of the resistance of the bridge wire in the ordinary manner.

R then forms our standard resistance, and is the quantity actually measured in the experiments. R is the resistance of a coil of platinum-silver wire made at Professor STUART's workshops, according to the pattern designed by Professor FLEMING, of Nottingham.

The wire was supplied by Messrs. ELLIOTT Brothers, and is that used by them in the manufacture of coils of about 100 units resistance. R, as has been stated, is about 160 units.

Now suppose that the connexions P P′, Q Q′ are broken, and that P Q is connected by means of one of the ┌┴┐ pieces. Break M N and connect L K, then the battery circuit is complete. Now connect M N again, the secondary circuit is complete, and the current running in the primary. Reverse the commutator C, an induction current is produced in the secondary circuit, and may be measured by the throw of the galvanometer needle.

To measure the primary current M N is broken and M L and H H′ are connected.

Between H and L the primary current is divided; the resistance in the direct circuit H V L being about 1 ohm, that in the circuit H S G Q P M L about 3072 ohms, so that about $\frac{1}{3073}$ of the whole current runs through the galvanometer; the permanent deflection of the galvanometer G is observed, and from this the value of the current is calculated.

We turn now to a detailed description of the apparatus used.

The coils were wound by Professor CHRYSTAL.

Two brass rings were carefully turned and a rectangular channel cut in the outer limb of each. A slit was cut in each ring to prevent currents in the frame (this was of course unnecessary for our experiments, but might render the coils more useful in many cases); the slit was closed with a piece of insulating material into which binding-screws connected with the wires of the coil are screwed.

The coils are lettered A and B.

In the experiments the coils are to be placed with their planes parallel and their centres on a line at right angles to their planes.

A cylindrical brass rod was carefully turned and fixed to a brass support so that it would stand in a vertical position.

Two tripod pieces of brass of the form shown in fig. 2 were constructed. The brass rod fits accurately through the aperture O in the centre of the tripod, and the plane of the arms O A, O B, O C is at right angles to the axis of the rod. The curved surfaces A A_1, B B_1, C C_1, are small portions of the same circular cylinder whose axis coincides with that of the rod, and the radius of this cylinder is the same as that of the carefully turned inner surface of the annulus on which the coils were wound.

This annulus thus would then fit on to A A_1, B B_1, C C_1 and rest on the flat surfaces A A_1 A_2 A_3, B B_1 B_2 B_3, C C_1 C_2, C_3, which are all parts of the same plane at right angles to the axis of the bar. If then we place one coil, A suppose, in this manner on the tripod, its plane is perpendicular to the axis of the bar, and its centre lies on that axis.

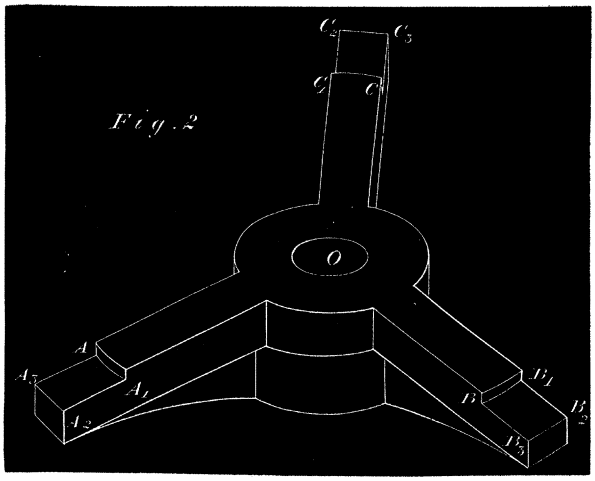


Fig. 2

To place the second coil with its plane parallel to the first we used three cylinders of brass, the ends of which had been carefully turned so as to be at right angles to their axes, while the lengths of the cylinders were as nearly as possible the same.

These three cylinders were placed vertically on the upper side of the coil A, one above each of the arms of the tripod, and the coil B rested on them ; its plane thus was as nearly as may be parallel to that of A.

To bring its centre into the axis a second tripod exactly similar to the first was placed in an inverted position over it, and the coil was moved about until the flange pieces fitted inside the inner surface of the annulus as before. In this manner the coils were adjusted to the required position.

The following account of the precautions used in the winding, and the methods employed to measure the constants of the coils, is quoted from a letter of Professor CHRYSTAL'S, addressed to one of the authors of this paper (R. T. G.) :—

"The coils, stand, &c., were constructed after working drawings made by myself under the supervision of Professor MAXWELL. The immediate end in view in constructing the coils was the determination of the ohm, and this of course influenced the design of the stand. It was proposed ultimately, to use the coils as a standard instrument for producing a uniform magnetic field in which to determine galvanometer constants and the like.

"The coils were wound by myself and the then mechanic at the laboratory, Mr. FULCHER. The coils were mounted for this purpose by placing between the three armed supports"—the tripods mentioned above—"which were then braced together and mounted on an axle and stand. During the winding constant tests were taken for the insulation between the wire and the metal channel. This was the main difficulty, and wherever the slightest defect was discovered the wire was unwound for a little way and paraffin paper and paraffin used. It was found absolutely necessary, in order to secure good insulation, to cover the bottom of the channel with a ribbon of silk drawn through melted paraffin. The number of turns in each layer was separately counted and registered, and as a check a counter was attached to the axle and read at the end of the windings; the two records agreed in both cases. The resistance of the coils after winding was 84·9 and 82·7 B.A. units respectively. Four diameters were measured in every layer.

"The measurements of length were made by means of the cathetometer, and I find in my book a record of a comparison between it and the beam-compass scale, which had been tested* I believe; there are also measurements of the thicknesses of the walls of the channel and its width."

The numbers actually used in calculating the value of M were furnished by Professor CHRYSTAL from these measurements. The error between the cathetometer and beam-compass appeared in one or two measurements to amount to 1 in 2000, but in the majority of cases it was very much less, its mean value being, perhaps, 1 in 10,000, irrespective of sign. On the whole then we may, without sensible error, treat the cathetometer scale as accurate.

In another letter, Professor CHRYSTAL gives the following extract from his note-book of the direct results of the observations. He says: "You remember that each diameter is given as the mean of four.

"The first diameter is through the slit, the next 45° from it in the direction of the sun's motion, the letter on the coil being up, and so on.

"Here is the entry in my book for the fourth layer in coil A.

No. of layer.	No. of turns.	Cathetometer.	Cathetometer.	Difference.	Mean.
4	26	781·40	280·14	501·26	
		781·30	279·95	501·35	
		781·27	279·58	501·69	
		780·60	279·52	501·08	501·35

"You will observe in the above extract that the two intermediate diameters are greater. This happens in most layers. At any rate the diameter perpendicular to that through the slit is in the great majority of cases the greatest, as might be expected."

Professor CHRYSTAL's measurements then gave us as the mean of four observations in different positions the value of the external diameter of each winding, and also the total number of windings in each layer. In each coil there were 30 layers and in each layer about 26 windings. In coil A the total number of windings was 797 and in coil B it was 791.

Let the external diameter of the layers be d_1, d_2, d_3, &c., and let the number of turns in a layer be $26+n_1$, $26+n_2$, &c.

* This beam-compass has again been tested by Mr. DODDS during the present year and found correct. All our measurements of length are referred to it. (Nov., 1882.)

Let t be the thickness of the wire and silk used, and let A and a be the mean radii of the coils.

$$\text{Then } 2A+t=\frac{26\{d_1+d_2+\ \ldots\ +\}+n_1d_1+n_2d_2+\ \ldots}{797}$$

Now we know the diameter of the channel before the first layer was put on and also the diameter of the first layer; they were respectively

49·565 and 49·728 centims.

From this we find

$$t=\cdot 0815 \text{ centim.}$$

and finally

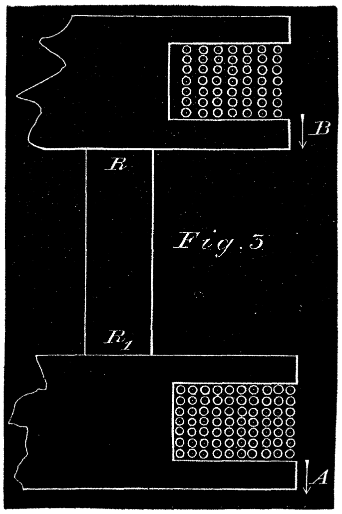
$$A=25\cdot 753 \text{ centims.}$$

The observations on coil B gave the same value for the thickness of the wire and covering, and we get for it

$$a=25\cdot 766 \text{ centims.}$$

The method here adopted to determine the value of the mean radius allows for the fact that in winding one layer may sink somewhat into the one beneath.

Let the figure (fig. 3) represent a section of the coils by a vertical plane through



the axis; let the coils be placed with their lettered sides down as in the figure. This we call position 1 throughout.

In position 2 the lettered side of B was turned uppermost.

In position 3 the lettered side of A also was uppermost, while in position 4, A remained uppermost while B was again inverted.

Thus if $\downarrow$B mean that the lettered side of B was down, we have

Position 1 $\begin{cases}\downarrow B \\ \downarrow A\end{cases}$ Position 2 $\begin{cases}\uparrow B \\ \downarrow A\end{cases}$

Position 3 $\begin{cases}\uparrow B \\ \uparrow A\end{cases}$ Position 4 $\begin{cases}\downarrow B \\ \uparrow A\end{cases}$

In the figure R R_1 represents a section of one of the cylindrical rods used to separate the coils; its length and the thickness of the walls of the channel are all known. If, then, we assume that the median plane of each coil is midway between the walls of the channel, we can find the distance between the median planes; this we denote by b. But if the median plane in one coil, B for example, be not halfway between the walls but some small distance, δb suppose, nearer the lower wall, then in position 1 the value assumed for b is too large by δb, while in position 2 it is too small by δb, and by making observations in positions 1 and 2 and taking their mean we eliminate any error in the result which depends on the first power of δb. Similarly, if there be an error in the position of the median plane of the coil A of the same nature it is eliminated by inverting A.

The thicknesses of the walls of the channel as given by Professor CHRYSTAL are:—

	Coil A.	Coil B.
	centim.	centim.
Lettered side	·478	·446
Unlettered side . . .	·488	·465

Also, if $2h$, $2k$ be the radial and axial dimensions of the channel for coil A, $2h'$, $2k'$ for coil B, we have

$$h=\cdot 96 \qquad k=\cdot 95$$
$$h'=\cdot 95 \qquad k'=\cdot 95$$

Three series of brass cylinders were used to separate the coils, and the lengths of these were measured each by two observers. A pair of calipers graduated to read with a vernier to $\frac{1}{1000}$th part of an inch were found to be the most convenient instrument with which to make the measurements. The scale of the calipers was tested against the beam-compass without discovering any errors that could affect the result to as much as 1 in 10,000.

The following is the series of measurements for the longest rods used (Series C), made by R. T. G. and given as an example of the agreement of the observations.

Calling the rods α, β, γ, we found

α.	β.	γ.
inches.	inches.	inches.
9·389	9·388	9·390
9·388	9·389	9·391
9·388	9·389	9·390

It was noticed, however, that two of these rods had been slightly bruised at the end, thus producing a small lump at one point; when the rods were placed so that this lump came between the jaws of the calipers an increase of ·002 inch was observed in the length. In the calculations this greater length has been used as the true length of these two rods. In the shortest rods, series A, the difference in length, arising from a slight lump on the end of one of the rods, was somewhat greater, being about ·003 inch.

Another series of measurements made in an entirely different manner by placing first the rods and then the beam-compass beneath a pair of reading microscopes gave very closely concordant results; these measurements made the rods appear about ·001 inch longer than the measurements with the calipers. Some difference of this kind was to be expected from the difference which exists between the contact length and the sight length of a rod.

In our calculations we have taken the values given by the calipers.

Reducing them to centimetres we have for the lengths of the rods

	centims.
Series A	12·182
Series B	15·416
Series C	23·856

We have thus obtained all the dimensions requisite for the calculation of the mutual induction between the coils in the three series.

Let us call b the distance between the mean planes; b, of course, is slightly different in each of the four positions included in each series.

The calculations of M have been conducted as follows:—

If all the windings are supposed to be coincident with the mean windings, and M_0 be the mutual induction on this hypothesis

$$M_0=4\pi nn'\sqrt{Aa}\left\{\left(c-\frac{2}{c}\right)F+\frac{2}{c}E\right\}$$

where

$$c=\frac{2\sqrt{Aa}}{\sqrt{(A+a)^2+b^2}} \text{ and } n, n' \text{ are the number of windings;}$$

and F, E are complete elliptic integrals to modulus c (MAXWELL, vol. ii., § 701).

Appendix i., ch. xiv., to the second edition of MAXWELL'S 'Electricity' contains a table in which the logarithms of $M_0/4\pi\sqrt{Aa}$ are given for values of $\sin^{-1} c$ from 60° to 90°, proceeding by intervals of 6′.

The proper value of $\gamma=\sin^{-1} c$ is most easily obtained from the equations

$$r_1^2=(A+a)^2+b^2 \qquad r_2^2=(A-a)^2+b^2$$

$$\cos\gamma=r_2/r_1.$$

Thus when the coils are separated by the long rods

$$r_1^2=3365{\cdot}64 \qquad r_2^2=712{\cdot}36$$

whence

$$\gamma=62^\circ\ 36'\ 32''.$$

From the table

$$\log \mathrm{M}_0/4\pi\sqrt{\mathrm{A}a}=\bar{1}{\cdot}5718069$$

and this gives, taking A as 25·750, a as 25·760

$$\mathrm{M}_0={\cdot}761225\times 10^8.$$

When, however, the dimensions h, k of the section are too large in comparison with b to allow us to mass all the windings together, we may use a formula of approximation due to Mr. H. J. PURKISS (Appendix ii., ch. xiv., MAXWELL, second edition).

Let the suffix $\pm h$ denote that for a is substituted $a\pm h$ in the corresponding functions, $\pm k$ that $b\pm k$ replaces b, and similarly for accented letters. Then

$$\mathrm{M}=\tfrac{1}{6}\{\mathrm{M}_h+\mathrm{M}_{-h}+\mathrm{M}_{h'}+\mathrm{M}_{-h'}+\mathrm{M}_k+\mathrm{M}_{-k}+\mathrm{M}_{k'}+\mathrm{M}_{-k'}-2\mathrm{M}_0\}.$$

To calculate these eight quantities in a methodical manner we notice that

$$\left.\begin{array}{l} r_{1h}^2=r_1^2+2(\mathrm{A}+a)h+h^2 \\ r_{2h}^2=r_2^2-2(\mathrm{A}+a)h+h^2 \\ \text{\&c.} \end{array}\right\}$$

and that consequently we have to add in each case a correcting square, and add or subtract a correcting product. The corresponding γ is found, and the rest of the calculation effected just as for the mean windings.

When the coils are separated by the long rods the greatest and least values of γ are $\gamma_{-k}=63^\circ\ 26'\ 54''$, and $\gamma_k=61^\circ\ 46'\ 56''$, the final value result being

$$\mathrm{M}={\cdot}761921\times 10^8.$$

It thus differs by nearly 1 in 1000 from the uncorrected value.

The values of b differ slightly according to the four positions of the coils, and a slight correction has to be made in the values of A, a assumed above.

It therefore becomes of importance to determine the correction in M to be made for the addition of ·001 centim. to A, a, or b.

Since $\cos\gamma=r_2/r_1$, it follows that

$$\delta_a\gamma=\delta_{\mathrm{A}}\gamma={\cdot}001(\mathrm{A}+a)/r_1^2\tan\gamma \qquad \text{in circular measure.}$$

This gives for the long rods

$$\delta_a\gamma=1{\cdot}64''.$$

Again

$$\delta_b\gamma = \cdot 001(r_1^2 - r_2^2)/r_1^2 r_2 \tan\gamma$$

whence

$$\delta_b\gamma = -3\cdot 16''.$$

From the tables we find that when $\gamma = 62°\ 36'$, the difference in the value of $\log M_0/4\pi\sqrt{a}A$ for an addition $1''$ to γ is 77×10^{-7}; that is, since $\delta M = \delta M_0$ very approximately,

$$\delta_a M/M = \frac{\delta a}{2a} + 77\times 10^{-7}\times 1\cdot 64''\times \log_e 10 = \cdot 000048$$

$$\delta_b M/M = -77\times 10^{-7}\times 3\cdot 16''\times \log_e 10 = -\cdot 000041$$

The true values of the radii are $A = 25\cdot 753$, $a' = 25\cdot 766$. The correction on this account to the value of M is therefore $9\times\cdot 000036\times 10^8 = \cdot 000324\times 10^8$.

The corrections to be applied to b are:—In position 1, $-\cdot 008$; in position 2, $+\cdot 016$; in position 3, $\times\cdot 006$; in position 4, $-\cdot 013$.

Making similar calculations for the medium and short rods we can present the values of the mutual induction thus:

TABLE giving values of the mutual coefficient of induction between the coils.

Position of coils.	Experiments A. Short rods.	Experiments B. Medium rods.	Experiments C. Long rods.
1	$1\cdot 55636\times 10^8$	$1\cdot 25797\times 10^8$	$\cdot 762107\times 10^8$
2	$1\cdot 55430\times 10^8$	$1\cdot 25649\times 10^8$	$\cdot 761321\times 10^8$
3	$1\cdot 55539\times 10^8$	$1\cdot 25727\times 10^8$	$\cdot 761735\times 10^8$
4	$1\cdot 55744\times 10^8$	$1\cdot 25875\times 10^8$	$\cdot 762521\times 10^8$
Mean of the four . .	$1\cdot 55587\times 10^8$	$1\cdot 25758\times 10^8$	$\cdot 761921\times 10^8$
Error in M produced by an error $\cdot 001$ in a or A . .	$\cdot 000063\times 10^8$	$\cdot 000052\times 10^8$	$\cdot 000036\times 10^8$
Error in M produced by an error $\cdot 001$ in b	$-\cdot 000108\times 10^8$	$-\cdot 000078\times 10^8$	$-\cdot 000041\times 10^8$

Thus the error produced by an error of ·001 centim. in b lies between ·005 and ·006 per cent., and the error in the measurement of b is certainly not more than ·001 inch or ·0025 centim.

Since the rods used to separate the coils were not exactly of the same length, the median planes cannot have been exactly parallel. The difference in the length of the rods is not as much as ·005 centim.

The radii of the coils are approximately 25 centims., and hence the angle between them is not as great as $\frac{\cdot 005}{37\cdot 5}$ or $\frac{1}{7500}$.

If we remember that when the coils are parallel the value of M is a maximum, so that the error due to the small angle between them depends on the square of the angle, it is clear that in our case this error is vanishingly small.

The galvanometer also was designed by Professor MAXWELL, and wound by Professor CHRYSTAL. It is also referred to in the article "Galvanometer," in the 'Encyclopædia Brittanica,' 9th edition. The description of it is taken from his account in the laboratory book, dated July, 1876.

There are two channels of rectangular section, and the following approximate dimensions:—

	inches
Depth of channel	$1\frac{1}{16}$
External diameter of bobbin	4
Breadth of channel	$0\frac{7}{8}$
Distance between channels	$0\frac{23}{32}$

Each channel contains 20 layers of thin copper wire and 16 layers of thick, making about 465 and 202 double turns respectively, so that there are 667 double turns in each channel, and about 2668 single turns on the galvanometer.

	inches.
Diameter of copper in thin wire	·014
Silk and all 82 thicknesses lie in	$1\frac{3}{8}$
Diameter of copper in thick wire	·029
Silk and all 34 thicknesses lie in	$1\frac{1}{6}$

The two thicknesses of wire were employed in order to fill the channels, and at the same time permit the resistance of the galvanometer to be reduced to the requisite amount. The ends of the wires are connected to binding screws on the bobbin marked A, B, &c., *a*, *b*, &c. A to *a* is one wire, B to *b* another. In our experiments the coils were connected up in series, the total resistance being about 60 ohms at a temperature of 13°·2 C.

The needle of the galvanometer was suspended from the WEBER suspension by three single cocoon fibres of 60 centims. in length.

The magnet was a small bar of hardened steel 1·5 centim. long, ·6 centim. broad, and ·12 centim. thick; its weight was ·708 grm. The magnet was attached by two small screws to a brass stirrup to which the mirror was fixed. A piece of brass wire 5·6 centims. long, with a screw thread cut on it, was fixed to this stirrup at right angles to the plane of the mirror, projecting equally on either side of the mirror. Two small brass cylinders could be screwed along this brass wire, and by means of them the moment of inertia and time of swing of the needle could be adjusted as required. The stirrup and mirror weighed 6·6 grms.

The galvanometer rested on a solid wooden base of about 18 centims. diameter, and this base was supported on three levelling screws.

A graduated circle is fixed to this stand, and the coils can be turned about a vertical axis, and their position read by means of a vernier. This was found useful in adjusting the coils parallel to the magnetic meridian. The galvanometer rested on a stone bracket built into the wall of the room. A scale placed approximately north and south at a distance of about 259 centims. from the magnet was reflected in the mirror and viewed through a telescope.

The scale rested on a solid wooden support on the floor of the room. The mirror, about 1·5 centim. square, was a specially good one, selected by a fortunate chance from among a number in the laboratory. The divisions of the scale were in millimetres, and after practice these could be subdivided by the eye with great accuracy to tenths. The scale itself was of paper; though this material is unsuitable for many purposes because of the changes produced by the weather in it, in our experiments these changes are of small consequence, for we require only the ratio of the throw produced by the induction current to the steady deflection produced by the permanent current; and the time which elapsed between the measurements was only a few minutes. Any shrinking or alteration of the scale will go on very approximately uniformly throughout its length and not alter the ratio of two lengths, which were never very unequal, as measured by the scale. After use the scale was carefully compared with the standard metre at the Cavendish Laboratory and the necessary correction applied to the readings.

The distance between the mirror and the scale only enters our result in the small correction necessary to reduce the scale readings so as to give the ratio of the sine of half the throw to the tangent of the deflection. It was unnecessary, therefore, to measure it with any great accuracy or to take steps to ensure its remaining the same from day to day; so long as it did not change during the half hour occupied by each experiment, all the conditions required by us were satisfied.

The resistance coils.

The standard coil R has been already referred to; the means adopted to measure its resistance will be described later.

Its value at a temperature of 14°·6 C. was found in May, 1881, to be 160·821 ohms.

The coil V used as a shunt to the galvanometer was made of thick German-silver wire. About 450 centims. of wire covered with silk were employed. The extremities of this were soldered to two stout copper rods with amalgamated ends, connexion with the rest of the apparatus being made by means of mercury cups; the ends of the rods were pressed down on to amalgamated pieces of copper at the bottom of the mercury cups.

The value of V was determined by repeated comparison at different temperatures with the B.A. unit known as "Flat coil" in Professor CHRYSTAL's report (Brit. Ass. Rep., 1876). The value of the Flat coil in mean B.A. units was taken from Professor FLEMING's recent comparison of the B.A. units at the laboratory. We have

Value of Flat coil at 15° C.	1·00003
Temperature of coefficient . . .	·00028 per 1° C
Value of V at 15° C.	1·0015
Temperature of coefficient . . .	·0003 per 1° C.

The coil itself is enclosed in a brass case and could be placed in a vessel of water. This was done during the experiment and the temperature noted by a thermometer graduated to fifths of a degree centigrade.

The coil S used with the galvanometer, when part of the battery current was sent through it, was a coil of platinum-silver wire of the ordinary form of about 3000 units resistance. It was immersed in the same vessel of water as V, and its temperature read by the same thermometer.

In our first series of observations the total value of the resistance M P Q G H, was observed and found to be 3072·38 ohms when the temperature of the coil S was 13·2.

The value of V at this temperature is 1·0011 ohms.

One extremity of the coil V dipped into the same mercury cup H as one extremity of the coil S, and the battery was also connected with this cup. The other extremity of V was connected by means of a piece of copper wire with L, the mercury cup in which the two portions of the battery current again united. This piece of copper wire was found to have a resistance of ·01556 ohm, so that the value of the resistance in the circuit H V L is 1·0167 ohms, at a temperature of 13°·2, and the currents in the two branches H V L and H G M L respectively, will be in the ratio of 3072·38 to 1·0167, so that if i be the battery current, that passing through the galvanometer will be $\frac{1{\cdot}0167}{3073{\cdot}39}i$.

T, as has been explained, was a variable resistance which could be adjusted so as to keep the difference between the resistance of the secondary circuit and the standard R sufficiently small to be measured in terms of part of the wire of the bridge E F.

During the experiments T had to be varied by somewhat over half a B.A. unit. Now T enters with S into the galvanometer circuit. The resistance, therefore, of this circuit was not quite the same during the observations, but varied by somewhat over ·25 ohm from its mean value, 3072·38 ohms.

The resistances W and U were two coils of about 30 ohms each wound on the same bobbin, and made of the same wire.

The galvanometer used with the WHEATSTONE'S bridge was one of about 150 ohms resistance, made by Professor STUART at the mechanical workshops, Cambridge.

Theory of the experiments.

Let $\overline{\mathrm{R}}$ be the absolute resistance of the secondary circuit including the galvanometer, M the coefficient of mutual induction between the coils, and i the current in the primary circuit, then the total induced current produced by reversing the primary is $\frac{2Mi}{R}$.

Let β be the first throw of the galvanometer needle produced by this reversal, T the time of a complete vibration, λ the coefficient of damping, G the galvanometer constant, and H the horizontal intensity of the earth's magnetism.

Moreover let τ be the coefficient of torsion of the suspending fibre.

Then,

$$\frac{2Mi}{R}=\frac{H(1+\tau)}{G}\frac{T}{\pi}\left(1+\frac{\lambda}{2}\right)\sin\frac{\beta}{2} \quad \dots \dots \dots \quad (1)$$

Again, let a current i' be passed through the same galvanometer directly afterwards, and let θ be the permanent deflection produced.

Then

$$i'=\frac{H(1+\tau)}{G}\tan\theta \quad \dots \dots \dots \quad (2)$$

Hence

$$R=\frac{2\pi M}{T\left(1+\frac{\lambda}{2}\right)}\cdot\frac{i}{i'}\cdot\frac{\tan\theta}{\sin\frac{\beta}{2}} \quad \dots \dots \dots \quad (3)$$

But

$$\frac{i}{i'}=\frac{S+V}{V}$$

Therefore

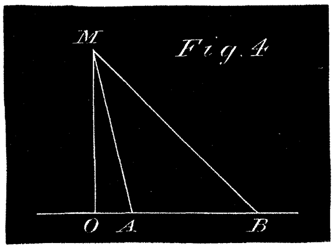
$$R=\frac{2\pi M}{T\left(1+\frac{\lambda}{2}\right)}\cdot\frac{S+V}{V}\cdot\frac{\tan\theta}{\sin\frac{\beta}{2}} \quad \dots \dots \dots \quad (4)$$

T is, of course, the observed time of oscillation.

The correction for the finite amplitude of the swing is too small to produce any error in the result. No correction for damping or torsion is required.

In the experiments the deflections on the scale were measured, not the angles of deflection. We require, then, to reduce the scale readings to angular measure.

Let p and q be the scale values of the throw and deflection, z the distance of the point of the scale vertically below the axis of the telescope from the point which appears to coincide with the cross wire when the needle is at rest. z being measured in the same direction as p and q, let a be the distance of the scale from the mirror.



Thus in the figure (fig. 4) let M be the centre of the mirror, O the point on the scale vertically below the axis of the telescope, A the point of the scale which appears to coincide with the cross wire when the needle is at rest, and B the apparent extremity of the throw.

Now the scale was carefully set so as to be at right angles to O M, and the distance O A was always small compared with O B.

Also

$$OA=z,\ AB=p,\ AMB=2\beta,\ OM=a.$$

Hence if we neglect squares of $\frac{z}{a}$ and fourth powers of $\frac{p}{a}$ and $\frac{q}{a}$

$$\text{angle } AMO=\frac{z}{a}$$

and

$$\tan\left(2\beta+\frac{z}{a}\right)=\frac{p+z}{a}$$

Therefore

$$\sin\frac{\beta}{2}=\frac{p}{4a}\left\{1-\frac{11}{32}\frac{p^2}{a^2}-\frac{pz}{a^2}\right\}$$

Also

$$\tan\left(2\theta+\frac{z}{a}\right)=\frac{q+z}{a}$$

hence

$$\tan\theta=\frac{q}{2a}\left\{1-\frac{q^2}{4a^2}-\frac{qz}{a^2}\right\}$$

and

$$\frac{\tan\theta}{\sin\frac{\beta}{2}}=\frac{2q}{p}\left\{1+\frac{11p^2-8q^2}{32a^2}+\frac{(p-q)z}{a^2}\right\}$$

Now the values of p, q, z, and a were such that $\frac{(p-q)z}{a^2}$ was about ·0001, we may therefore write with sufficient accuracy

$$\frac{\tan\theta}{\sin\frac{\beta}{2}}=\frac{2q}{p}\left(1+\frac{11p^2-8q^2}{3^2}\right)$$

p and q being the scale values of the throw and deflection.

To observe these quantities accurately the following adjustments are necessary.

The scale should be parallel to the mirror when at rest.

The coils of the galvanometer should be north and south so that their plane may be parallel to the magnetic axis of the needle.

The telescope should be placed so that its axis and the normal to the mirror, when at rest, may be in the same vertical plane. If this is the case the division of the scale which appears to coincide with the vertical cross-wire will be that just below the axis of the telescope, supposing, as in our case, the telescope is placed so as to look just over the edge of the scale.

In making these adjustments the scale and telescope were first fixed so that division 250—the middle of the scale—was vertically below the centre of the object-glass of the telescope; the galvanometer was placed in position and levelled, and the telescope adjusted to view the image of the scale in the mirror.

Then telescope and scale were both moved until the division of the scale which coincided with the cross-wire was close to 250. When this was the case the normal to the mirror and the axis of the telescope were nearly in the same vertical plane.

The scale was then turned in a horizontal plane until its two ends, equidistant from division 250, were also equidistant from the mirror, taking care at the same time that the image of division 250, the centre of the scale, remained close to the cross-wire of the telescope.

The scale was thus put at right angles to the normal to the mirror.

A long bar magnet resting on a pivot at its centre was then supported close to the scale, and it was found that the scale was very nearly parallel to the axis of the magnet, the error was certainly not more than 20′.

Thus the scale has been set very approximately north and south, and since the mirror is very nearly parallel to the scale, it is also nearly parallel to the axis of the galvanometer needle. Hence, if we set the coils parallel to the scale or mirror they will be very nearly north and south, and their plane will be approximately parallel to the axis of the needle.

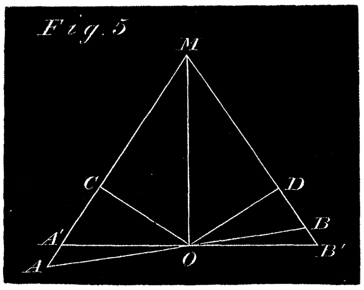
A piece of plate glass can be screwed on to the galvanometer in a position very nearly parallel to the coils. This was done, and the reflection of a lamp placed just below the telescope was observed, the galvanometer coils were turned until this reflected image was seen in the centre of the field of the telescope. Thus the galvanometer coils were placed very nearly north and south. The reading of the vernier attached to the galvanometer was noted, and by means of it the coils could readily be brought back to the same position, or placed at any required angle to the meridian.

The adjustments thus described were of course only approximate, but it is easy to show that the method of experiment eliminates any small outstanding error.

Let us suppose the coils are inclined at an angle α to the meridian. The effect of this is merely to change G into G $\cos\alpha$ in both the equations (1) and (2), and $\cos\alpha$ disappears from the resulting equation. It is better, however, that the field of force produced by the current in the galvanometer coil should be as nearly as possible uniform throughout the space through which the needle moves in the throw and deflection respectively. This condition is best satisfied if the needle when in equilibrium is parallel to the coils.

Let us now suppose that the scale is inclined at an angle γ to the plane of the mirror. Let p_1 and p_2 be the scale values of the throw to the right and left of the resting point.

Let A O B (fig. 5) be the scale, O M the normal to the mirror, draw A′ O B′ at right angles to O M.



Then A O $A'=\gamma$; let A and B be the observed extremities of the throw or deflection; θ its value in angular measure; let M A, M B, cut A′ B in A′ and B′, and let p be the true scale value of the deflection, then

$$A'O=OB'=p$$
$$AOM=MOB=2\theta$$

Draw O C, O D perpendicular to M A, M B respectively.
Then

$$OC=OA'\cos COA'=OA\cos COA$$

Thus

$$p\cos 2\theta=p_1\cos(2\theta+\gamma)$$

Similarly

$$p\cos 2\theta=p_2(\cos 2\theta-\gamma)$$

Therefore

$$2p=(p_1+p_2)\cos\gamma-(p_1-p_2)\tan 2\theta\sin\gamma$$

Now p_1 was always very nearly equal to p_2, and 2θ and γ are both small.
Hence very approximately indeed we have

$$p=\frac{p_1+p_2}{2}\cos\gamma$$

Similarly

$$q=\frac{q_1+q_2}{2}\cos\gamma$$

Thus, by observing deflections right and left and taking the mean, we get a value for the ratio of p/q which is independent of an error in the azimuth of the scale, much greater than anything possible in the actual experiments.

If the magnet be not parallel to the mirror the angle turned through by the mirror is still that turned through by the magnet; the fact that the magnet and mirror were very nearly parallel afforded a ready means of setting the plane of the coils in the magnetic meridian.

The time of a complete vibration was measured in the usual way by noting the times of 8 or 10 transits of the resting point over the cross wire of the telescope, then waiting for the period occupied by some 10 or 12 oscillations and again observing the times of 8 or 10 transits. The value thus determined requires reducing to that for an infinitely small arc.

Now we know that if during the observation the arc of oscillation change from α_1 to α_2, and if T′ be the observed time of oscillation, then

$$T=T'\left\{1-\frac{1}{8}\left(\sin^2\frac{\alpha_1}{4}+\sin^2\frac{\alpha_2}{4}\right)\right\}$$

neglecting higher powers.

In the observations the value of α_1 was about 3°, that of α_2 about 1°·30′, the correction thus amounts to ·000025 and is quite inappreciable.

The value of λ was obtained by setting the needle vibrating, the secondary circuit being closed, and observing a series of resting points. If p_1, p_n be the amplitudes of the first and n^{th} vibration we have

$$\lambda=\frac{1}{n-1}\log_e\left(\frac{p_1}{p_n}\right)$$

Two independent observations of 17 vibrations gave

$$\frac{p_1}{p_{17}}=1·2913 \text{ and } 1·2909$$

Whence

$$\lambda=·0159$$

$\bar{R}$ is the absolute resistance of the secondary circuit; this is very nearly but not quite equal to R, the resistance of our standard coil, and the difference between the two can be expressed in terms of the resistance of the wire of the B A bridge. This wire is 1 metre in length, and is divided into millimetres; let ρ be the resistance of 1 millim. The wire is graduated from E to F, (fig. 1); let G_1 be the position of the sliding contact piece when there is no current through the galvanometer, P P′, Q Q′, and M N being connected.

Let

$$EG_1=x \text{ millim.} \qquad FG_1=y \text{ millim.}$$

$\bar{R}$ is the resistance of the circuit Q′ Q G B N M P P′.

Hence

$$\frac{\bar{R}+x\rho}{R+y\rho}=\frac{W}{U}$$

Now interchange U and W, and let x' y' be the new values of x and y.

Thus

$$\frac{\overline{R}+x'\rho}{R+y'\rho}=\frac{U}{W}$$

But

$$x+y=x'+y'$$

Hence

$$\overline{R}+x'\rho=R+y\rho$$

$$R=\overline{R}+(x'-y)\rho$$

A number of experiments were made to determine ρ, and the value $\rho=\cdot000072$ B. A. unit was obtained.

Since ρ only comes in as a small correction, we may take one B. A. unit as 1 ohm.

Again, the value of R depends on the temperature, and our experiments required reducing to a constant temperature t_0; let t be the temperature of R at the time of experiment, R_0 the value of R at temperature t_0, and α the coefficient of increase of resistance per degree centigrade. Then we have

$$R=R_0\{1+\alpha(t-t_0)\}$$

Hence, finally our equation (4) becomes

$$R_0\{1+\alpha(t-t_0)\}=\frac{2\pi M}{T\left(1+\frac{\lambda}{2}\right)}\cdot\times\frac{S+V}{V}\times\frac{2(q_1+q_2)}{(p_1+p_2)}\left\{1+\frac{11p^2-8q^2}{32a^2}\right\}+(x'-y)\rho \quad . \quad . \quad (5)$$

The experiments were made in the following order :—

The time of swing was observed, the secondary circuit being closed as in the experiments.

The variable resistance in the secondary circuit was adjusted until the difference between $\overline{R}$ and R could be measured in terms of the bridge-wire resistance, and the values of x y, x' y' determined. While this was being done a second observer read the temperatures of the coils R, S, and V, and the galvanometer G.

The connexion P P′, Q Q′ were broken, and P Q was joined. The resting point of the reflected image of the scale was observed when no current was passing through the galvanometer. This was done in the usual manner by observing five consecutive turning points.

The galvanometer needle was brought as nearly as possible to rest by the use of a damper. This consisted of a coil of wire placed near the needle, through which the current from a single LECLANCHÉ cell could be passed. By means of a second key a shunt could be introduced into this circuit so as to allow only a small fraction of the current from the battery to circulate in the coil. After a little practice the apparent oscillations of the galvanometer needle could easily be reduced to a few tenths of a millimetre of the scale.

When this had been accomplished, the battery current in the coil A was reversed and the first throw of the galvanometer needle observed. Suppose this was a throw to the right. By reversing the current at the right instant the needle could be brought very nearly to rest again, and the small swing that remained was easily destroyed by the damper. The connexions were then adjusted so that on again reversing the battery current a throw to the left was observed. A second throw to the left was observed and then a second throw to the right.

After this another reading was taken of the resting point when no current was passing.

The connexions were then altered so that a fraction of the direct current could be passed through the galvanometer, and the position of rest of the needle when the current was passing was observed. The needle was easily brought sufficiently nearly to rest in its new position by making the primary contact for a third of the time of swing, then breaking it for a second third, and finally making it again and leaving it made.

The resting point was determined while the needle was swinging from the observation of five consecutive turning points. The primary current was then reversed and a deflection in the other direction observed. After this a third reading of the resting point without any current was taken.

The connexions were again altered to observe the throws and four more were taken, one to the left, two to the right, and one to the left. A fourth observation of the resting point completed this part of the observations. The secondary circuit was then put into communication with the WHEATSTONE's bridge, and the difference between R and $\bar{R}$ measured, giving a second series of values of x y, x' and y', and finally the thermometers were all read again.

When we had become accustomed to the work a complete set of observations, excluding the time of swing, took about 25 minutes.

In the first series of measurements the time of swing was only observed twice each afternoon—at the beginning and end of the afternoon's work. In the second series it was taken generally at the beginning of the work and after every second or third set of observations, that is to say, at intervals of somewhat less than an hour and-a-half.

Throughout the experiments one observer (R. T. G.) read the galvanometer deflections, while the other (J. M. D. in the first series, E. B. S. in the second) made or broke the various connexions as required, and noted down the scale readings as they were read out by the observer at the telescope.

To obtain from the direct results of the observations the quantities required for substitution in formula (5), the following method was adopted :—

The means of the temperatures at the beginning and end of the observation were taken as the temperatures at the time of the experiment, and the means of the values of $x\ y$, $x'\ y'$.

The alteration in temperature during the experiment was only about ·1° or ·2° C. The changes in the values of $x\ y$, &c., during the observation were produced by the variation of the temperature of the secondary coil, which being copper has a high temperature coefficient. The alteration in the value of x was rarely as much as 300 bridge divisions, and since this change went on nearly uniformly during the experiment, the mean of the two values at the beginning and end will be very accurately the true value.

In reducing the scale readings to give the throws and deflections, we had to remember that owing to the variation in the direction of the earth's magnetic force, there was a continual change going on in the resting point of the needle.

In general, this change was only ·3 or ·4 millim. during the time occupied by a set of observations, sometimes it amounted to 1 millim. or rather over, and on one or two occasions during magnetic storms the changes were so violent and sudden that we had to cease work entirely.

For determining the value of the throw the following method enables us to eliminate the effect of this change in zero.

Take the mean of the two throw readings to the right, then the mean of the two to the left, and the mean of the resting point readings, then the differences between the throw readings and the resting point, or zero readings, will give the throws right and left respectively, corrected for change in resting point. The difference between the throw readings will give the value of p_1+p_2 directly; since, however, the throws right and left ought to be the same if the adjustments are correct—it forms a test of the accuracy of the measurements to calculate p_1 and p_2 separately.

For the permanent deflection, however, in which only one observation was made on either side of the zero, the same method is not applicable. The four observations we have to consider are: zero reading, deflection to right, deflection to left, zero reading.

Suppose the zero is moving from right to left, then, if we take the mean of the zeros and consider the differences between it and the deflection reading as the deflections right and left, in each case our deflection will be too great, while, if the zero be moving in the other direction, the deflection obtained will be too small.

To obviate the difficulty we assumed that the interval of time between each two consecutive observations was the same, and that the change in zero was uniform. We then obtained by interpolation the values of the zero readings at the moments of making the deflection observations. The differences between these and the deflection readings gave then the true deflections right and left, q_1 and q_2 respectively, the whole correction being a very small fraction of the measured deflection.

The second series of throws were then treated in the same manner as the first, and a second pair of values of p_1 and p_2 obtained. These generally differed somewhat from the first, for the electromotive force of the battery—a combination of DANIELL'S cells

—was not absolutely constant throughout an experiment. Now the observations of deflection refer to a moment of time about half way between the two throws, so that the mean of the two values of p_1+p_2 will give us the value of that quantity corresponding to the value of q_1+q_2 obtained for the deflections.

For the first series of observations the difference in the values of p_1 before and after the deflection reading was sometimes, but not often, as great as 1 millim. in a throw of about 215 millims.

A copy of the observations requisite for one experiment will perhaps render the above details more complete.

OBSERVATION No. II., June 8th, 1881, 11·45 a.m. Observers, R. T. G., J. M. D.
Time of swing 23″·277.

Bridge reading value of x	U, W direct		500
,, ,, x'	U, W interchanged		410

Temperature.

R 13°·7 S 13°·7 Galv. 14°·5

Scale observations.

Zeros	235·5	
		227·5
	235	
		227·8
	235	
	Left.	Right.
Throws	7·0	
		455
		454·5
	7·8	
Zeros	225	
		238
	225·2	
		238
	225·2	

	Left.		Right.	
Deflections	87·2		364·8	
		94·5		377
	87·5		364·8	
		94·6		377
	87·8		365	

Zeros	237·8	
		224·2
	237·5	
		224·5
	237	

	Right.	Left.
Throws	454·2	
		6·0
		6·5
	454	

Zeros	234·5	
		227
	234·2	
		227
	234	

Temperatures.

R 13°·7 S 13°·8 Galv. 14° 8

Bridge reading value of x { Direct } 630
" " x' { Interchanged } 540

Battery: four ordinary DANIELL'S in series.

From these observations we obtain the following value for the zero and deflection readings :—

Zeros	231·4	231·5	230·8	230·6

	Left.	Right.
Deflections	91·0	370·9

Thus the mean zero for the first throw is 231·45, and combining this with the observations of the throw we get

$$p_1 = 454 \cdot 75 - 231 \cdot 45 = 223 \cdot 3$$
$$p_2 = 231 \cdot 45 - \quad 7 \cdot 4 \ = 224 \cdot 05$$

While for the second throw the zero is 230·7, and the values are

$$p_1 = 454 \cdot 1 - 230 \cdot 7 \ = 223 \cdot 4$$
$$p_2 = 230 \cdot 7 - \quad 6 \cdot 25 = 224 \cdot 45$$

Thus the mean value of p_1 for the experiment is

223·35

and of p_2

224·25

These numbers require correcting for scale error.

The correction to p_1 is

+1·2

that to p_2 is

+1·0

so that

$$p_1 = 224 \cdot 55 \text{ millims.}$$
$$p_2 = 225 \cdot 25 \text{ millims.}$$

The difference between them being only ·7 millim. it is clear that the adjustments are all right.

Before taking the deflections the zero reading was 231·5, after taking them it had become 230·8. Thus interpolating, the zero reading at the moment of the deflection to the left was 231·3, and we find

$$q_2 = 231 \cdot 3 - 91 \cdot 0 = 140 \cdot 3$$

while at the moment of the deflection to the right the zero reading was 231·1, and, hence,

$$q_1 = 370 \cdot 9 - 231 \cdot 1 = 139 \cdot 8$$

Correcting for scale error we have

$$q_1 = 140 \cdot 6 \text{ millims.}$$
$$q_2 = 141 \cdot 1 \text{ millims.}$$

We have also

Mean value of temperatures—

R 13°·65 S 13°·7 Galv. 14°·65

Mean value x . . . 565

" x' . . . 475

We proceed now to give in tabular form the results of our first series of experiments made in June, 1881.

The table will contain the values of $\frac{p_1+p_2}{2}$, $\frac{q_1+q_2}{2}$, the temperature of R, the mean values of x and x', and the time of swing. The corrections introduced by the variation in the temperature of S and the galvanometer we shall show, when we come to discuss the results, are so small that they may be neglected. We may, therefore, treat the results throughout as if the temperature of S were 13°·2, and that of the galvanometer 14°.

The battery used in all cases was four DANIELL'S cells.

The average difference between p_1 and p_2, irrespective of sign, was about ·32 millim., in one case only was it as great as ·9 millim.; it is hardly necessary, therefore, to give both p_1 and p_2 in the table.

The rods used to separate the two coils, primary and secondary, were the same, but, as has been explained, the coils were placed in each of four positions, numbered respectively I., II., III., and IV.

TABLE I.

Position.	Mean throw.	Mean deflection.	T.	Temperature R.	x.	x'.
I.	226·10	142·15	23″·277	15°·1	670	590
	226·25	142·40		15·2	583	500
	223·00	140·25		15·1	420	350
	222·70	139·70	23·250	15·2	540	470
	224·55	141·40	23·277	13·8	690	600
	224·90	140·85		13·7	440	355
II.	223·65	140·30		13·7	750	665
	222·80	140·20		13·9	505	420
	222·25	139·85		13·9	265	175
III.	218·00	137·05		14·2	770	670
	217·60	136·90		14·2	480	390
	217·50	136·70		14·3	325	240
IV.	216·05	135·85		14·3	778	680
	216·10	135·55		14·5	532	440
	215·85	135·50	23·274	14·5	400	310

The first four observations were made on June 7th, 1881, the last eleven on June 8th.

For the first four observations, therefore, we take the time of swing as 23″·264, for the last eleven as 23″·275.

The mean temperature of R during the experiments is about 14°·6; we take this then as the value of t_0. The temperature coefficient of R—a platinum-silver coil—may for the small range considered be taken as ·0003 per ohm per degree.

Thus the values of the constants in the formula (5) for R_0 are

$$t_0=14^\circ{\cdot}6 \qquad \alpha={\cdot}0003$$
$$\tau={\cdot}0007 \qquad \lambda={\cdot}0159$$
$$a=259 \text{ centims.}$$
$$\rho={\cdot}000072 \text{ ohm.}$$
$$\frac{S+V}{V}=\frac{3073{\cdot}39}{1{\cdot}0167}$$

while giving p and q their mean values for the whole series of experiments, the term

$$\left\{1+\frac{11p^2-8q^2}{32a^2}\right\} \text{ comes to } 1{\cdot}0018$$

As we have explained, the value of M is slightly different for each of the four positions.

The methods used to determine have been explained ; the values obtained were

$$M_1=1{\cdot}55636$$
$$M_2=1{\cdot}55430$$
$$M_3=1{\cdot}55539$$
$$M_4=1{\cdot}55744$$

where the suffixes refer to the positions.

Table II. gives the results of the calculations. As the results stand there, the negative errors are fewer in number than the positive, the two greatest errors being negative.

The greatest of these is ·453, which is about 1 part in 350.

TABLE II.

Position.	R_0 in ohms.	Error from mean of set.	Mean value of R_0 for each position.	Error from mean of whole.	Percentage error.
I.	158·82	·147	158·673	·047	·029
	158·98	·307			
	158·84	·167			
	158·48	—·193			
	158·70	·037			
	158·22	—·453			
II.	158·27	—·297	158·567	—·059	—·039
	158·68	·103			
	158·76	·193			
III.	158·69	·093	158·597	—·029	—·018
	158·77	·173			
	158·33	—·267			
IV.	158·87	·241	158·629	·003	+·002
	158·51	—·119			
	158·51	—·119			

Mean value . . . . 158·626 ohms.
Mean error of mean of each from mean of whole . . ·037
Mean percentage error . . . . ·023

We shall retain the whole series of observations and take as the resistance of our standard coil R at a temperature of 14°·6 C., the value

$$158{\cdot}626 \frac{\text{earth quadrants}}{\text{second}}$$

It remains now to explain the method used to determine the values of the resistances.

For this purpose the coils in a post office resistance box, made by Messrs. ELLIOTT Brothers, were compared with the standards at the Cavendish Laboratory. The 1 unit coil of the box was compared with the coil known as Flat in Professor CHRYSTAL'S report, then the 1-unit + Flat were balanced against the 2-unit coil of the box, then this 2-unit against the second 2-unit, which we will denote by 2′, then 1+2+2′ against the 5-unit coil, and so on.

In this manner all the coils between 1 and 2000 B.A. units were compared.

The British Association wire bridge was used in making the comparison.

In the ordinary use of this (CAREY-FOSTER'S method) the two coils to be compared are connected to the ends of the bridge wire and a measurement taken, the coils are then interchanged and another observation is taken, and from these two the difference between the coils is expressed directly as the resistance of a portion of the bridge wire. We, however, could not apply this method, for, calling P and Q the coils to be compared, since P and Q are coils in the same box, one end of P is always in electrical connexion with one end of Q. The following arrangement therefore was adopted:—

Two coils of known resistance were connected one to each end of the bridge wire, while P and Q formed the other arms of the bridge. The coils actually used were those marked F and G in CHRYSTAL'S report.

The sliding contact was adjusted till no current passed through the galvanometer, and its position noted.

Let $\alpha+x$ be the resistance of the portion of the wire connected with F, $\alpha-x$ of that connected with G, so that 2α is the whole resistance of the bridge wire. Let $1+\delta F$, $1+\delta G$ be the resistances of F and G at the temperature of the observation.

δF and δG are very small.

At 14°

$$\delta F = -{\cdot}00084 \text{ ohm}$$
$$\delta G = -{\cdot}00112 \quad ,,$$

Then we have

$$\frac{P}{Q} = \frac{1+\delta F+\alpha+x}{1+\delta G+\alpha-x}$$

Interchange F and G and let x' be the new value of x

$$\frac{P}{Q} = \frac{1+\delta G+\alpha+x'}{1+\delta F+\alpha-x'}$$

Whence $x+x' = \delta G - \delta F$.

Thus x and x' are exceedingly small, and if we neglect squares and higher powers of δF, δG, x, and x', we obtain

$$(1+\alpha)\frac{P-Q}{Q}=x-x'$$

Now $\alpha=\cdot036$ mean B.A. unit.

If then we know Q, the value of P can be found from the observations of x and x'.

Two series of observations were taken, one by R. T. G. the other by J. M. D., each observation in each series being the mean of 2 or 3.

The extreme difference between the two series was in no case more than 1 in 3000.

The box remained in the room for some time before taking the observations, and the temperature was supposed to be that of the room as indicated by a thermometer laid on the box. A small correction was made for the resistance of the copper rods which connected the box to the bridge and the plugs in the box. This was determined by one observer (R. T. G.) only, so that any error in it will affect both measurements equally. We shall show, however, shortly that it cannot affect the value of the B.A. unit as determined from our measurements. Having thus obtained the values of the resistances in the box in terms of the B.A. standard units, the value of R_0 in these units was determined by the ordinary method. We found thus the mean of several closely concordant measurements

$$R_0=160\cdot821 \text{ B.A. units}$$

the temperature being $14^\circ\cdot6$ C.

The resistance S of our secondary circuit and galvanometer was determined in terms of the coils in the same box, and we found

$$S=3072\cdot38 \text{ B.A. units}$$

at a temperature of $13^\circ\cdot2$ C.

Owing to the difficulty of determining the value of the resistance of the plugs in the box and the copper connecting pieces, either of these results may possibly have an error of about 1 in 1500. Now the value of S enters into the value of R_0 in absolute units and affects it in the same way.

To determine the value of the B.A. unit we require the ratio

$$\frac{R_0\dfrac{\text{earth quadrant}}{\text{second}}}{R_0 \text{ in B.A. units}}$$

Its value is

$$\frac{158\cdot626}{160\cdot821}\text{ ohms.}$$

Whence we get

$$1 \text{ B.A. unit}=\cdot98635 \text{ ohm}$$

But the error we have been considering affects in exactly the same manner the numerator and denominator of this ratio. If in consequence of it one is too great so also is the other in the same proportion. Thus the accuracy of our result is not impaired by the uncertainty of this correction. In fact, although to determine the resistance of our coil R in absolute measure we require to determine accurately a ratio of 1 to 3000, and this determination has generally been held to be one of the main objections to our method, yet to determine the value of the B.A. unit we have in addition to compare a ratio of 160 to 1. Thus, in fact, to determine the value of the B.A. unit the ratio to be compared is 160 to 3000, or about 1 to 19, and this is a much easier experiment to make.

Our 160 and 3000 have both been expressed in terms of the resistances of the box, and even though there may be some considerable error in the actual values of these resistances, the error in the ratio of any two of them is a quantity very small indeed.

In conclusion, we would refer to another objection which has been made to the method. Nearly the whole of the battery current is allowed to flow through the coil V, whose resistance is about 1 ohm; the effect of this must be to heat V and alter its resistance, thus producing error. We shall show that the error in our case is vanishingly small.

The electromotive force of the battery was at most about 5 volts, and the total resistance of the primary circuit was about 80 B.A. units. The coil V was of German-silver wire, about 450 centims. being used to make it; the wire thus was very thick, its radius being ·06 centim. The wire, silk covered, was loosely wound in a coil and enclosed in a brass case, which was immersed in water.

From these data we find that the amount of heat developed per minute in the coil will be ·055 unit.

If we suppose all this heat to be retained, the rise of temperature will be ·015° C. per 1′, and the increase of resistance ·0000045 B.A. unit, and this will be too small to affect our results. As a matter of fact, it is clearly impossible for all the heat to remain in the coil, and the correction is, *à fortiori*, too small to be considered.

During the second series of experiments a hole was bored in the brass case of the coil and a thermometer inserted. The thermometer agreed throughout in its readings with that in the water bath in which the coil was immersed.

Thus we conclude as the final result of this series of experiments that the value of the B.A. unit is ·98635 ohm.

The agreement between the individual experiments of the series is remarkable. They are, however, open to the objection that the conditions under which they were taken remained unaltered in some essential particulars. Thus the rods used to separate the primary and secondary coils were the same throughout, while the battery was also the same. It was decided, therefore, to make a second series of observations in which these quantities were varied. This was done during November and December, 1881. Mr. DODDS had left Cambridge, and his place was taken by Mr. E. B. SARGANT, of Trinity College.

PART II.

Profiting by our past experience, the arrangement of the apparatus was modified slightly.

In Part I., as has been explained, one electrode of the coil V dips into the mercury cup H, while the other is connected by means of a piece of copper wire with the cup L.

In Part II. the mercury cups and resistance coils were so placed that the second electrode of V dipped directly into the cup L; the piece of copper wire between the two, therefore, was dispensed with.

Again, in fig. 1 it will be seen that between P and M in the secondary circuit there is a variable resistance T, used to adjust the resistance of the circuit until it balanced R.

This variable resistance formed a part of one of the two paths open to the current when measuring the deflection, and, as we have said, renders the exact proportion into which the current was divided in each experiment uncertain to the amount of about 1 in 6000.

In fitting up the apparatus for Part II., T was placed between B and N. As before, the resistance of the secondary circuit could be adjusted, but that of the primary remained unaffected by alterations of T, which in this part formed no portion of it.

Three sets of rods were used to separate the primary and secondary coils, we shall call them A, B, and C, respectively. The rods A were those used in Part I.

With the rods A three different electromotive forces were used; the batteries employed being respectively four ordinary DANIELL'S, two ordinary DANIELL'S, and five THOMSON'S tray DANIELL'S.

In position B we had five THOMSON'S-DANIELL'S, and in position C five THOMSON'S-DANIELL'S and six THOMSON'S-DANIELL'S.

As before, the coils were placed in positions I., II., III., and IV., but the order of taking the observations was somewhat varied. In Part I. three observations were taken in each position without altering the coils; in Part II., however, after taking one observation in position I., one of the coils was reversed so as to bring them into position II., and an observation made; the other was then reversed, and so on, and after the four measurements had been taken the whole series was repeated. This method necessitated rather more handling of the coils than the other; it had, however, the advantage that each set of four observations was taken under more nearly similar conditions, while, in consequence of the more frequent setting of the coils, the error due to any one chance bad setting was reduced. The time of swing was observed more frequently, being taken twice and generally three times for each set of four. The times corresponding to the mean throw and deflection are given in the table, being obtained by interpolation from those actually observed.

Table III. gives the same observations for Part II. as are given in Table I. for Part I.

With reference to the table we must notice that the experiments were not always made in the order I., II., III., IV., and therefore the times of swing in the column headed T are not in order of magnitude. Thus, in the second set, the real order of the experiments was II., I., III., IV., and in this order the values of T increase uniformly by 0″·005. In the experiments with two DANIELL'S, experiment IV. was made on November 16th, experiment II. on November 18th, and experiments III. and I. on November 21st.

TABLE III.—Series A.

Battery.	Position.	Mean double throw.	Mean double deflection.	T.	Temperature R.	x.	x'.
4 DANIELL'S	I.	359·1	223·35	23″·384	12°·4	495	400
	II.	360·4	224·5	23·384	12·7	640	555
	III.	362·75	225·2	23·386	12·8	630	540
	IV.	361·9	225·3	23·388	13·1	615	525
4 DANIELL'S	I.	364·2	227·35	23·415	13·4	350	270
	II.	364·55	227·7	23·410	13·4	300	215
	III.	359·4	223·8	23·420	13·3	615	530
	IV.	352·45	220·2	23·425	13·0	565	455
5 THOMSON'S	I.	451·2	280·8	23·373	11·9	510	420
	II.	453·2	281·9	23·367	11·8	420	330
	III.	452·2	280·4	23·380	12·2	525	435
	IV.	450·9	280·8	23·378	12·1	515	430
2 DANIELL'S	I.	190·55	119·1	23·386	12·4	575	490
	II.	191·8	119·8	23·405	13·4	415	335
	III.	190·75	118·8	23·382	12·3	805	715
	IV.	191·3	119·6	23·391	13·4	570	490

TABLE III.—Series B.

Battery.	Position.	Mean double throw.	Mean double deflection.	T.	Temperature R.	x.	x'.
5 THOMSON'S	I.	374·5	287·95	23″·344	11°·6	630	545
	II.	373·25	286·5	23·355	11·9	895	800
	III.	375·1	288·3	23·322	10·8	415	335
	IV.	374·15	286·9	23·333	11·2	780	700
5 THOMSON'S	I.	370·9	285·6	23·406	12·6	575	485
	II.	372·0	286·3	23·405	12·5	465	385
	III.	369·15	285·15	23·407	13·0	480	380
	IV.	370·5	286·2	23·406	12·8	505	415

TABLE III.—Series C.

Battery.	Position.	Mean double throw.	Mean double deflection.	T.	Temperature R.	x.	x'.
5 THOMSON'S	I.	227·4	289·8	23″·379	12·4	560	420
	II.	229·6	292·4	23·382	12·5	655	575
	III.	228·0	290·7	23·384	12·8	705	620
	IV.	228·2	290·5	23·381	13·0	690	605
6 THOMSON'S	I.	252·9	321·9	23·389	13·6	555	485
	II.	253·1	323·4	23·391	13·3	545	475
	III.	254·7	323·9	23·393	13·2	445	340
	IV.	250·2	318·9	23·395	12·9	425	335
6 THOMSON'S	I.	243·25	310·95	23·381	13·4	950	860
	II.	242·8	309·8	23·374	12·7	680	590
	III.	240·75	307·1	23·392	14·0	915	830
	IV.	240·65	306·85	23·384	13·8	280	190

These direct experimental results require to be substituted in our formula in order that we may obtain the values of R_0. The coils were placed in the four positions I. to IV. in order to eliminate any small unknown error in the position of the mean plane.

Now our first series of experiments have been sufficient to show that this error must be exceedingly small, and the result obtained by taking the mean of the four will certainly eliminate it.

Instead, therefore, of giving the value of R_0 for each position, we shall only calculate the mean value for each set of experiments, using, of course, as our value of M, the mean of the values M_1, M_2, M_3, M_4.

The correction for damping is the same throughout the whole series of observations, the value of $1+\frac{11p^2-8q^2}{32a^2}$, of course, differs for the different currents used; it therefore is included in the table.

The value of M for each of the three series A, B, C, is also given in the table.

The value of λ found from a large series of closely concordant measurements was

$$\lambda = \cdot 01368$$

In these experiments the temperature was somewhat lower, generally, than in Part I., so that the mean value of the temperature of R was about 12° and $t_0=12°$.

The distance between the mirror and the scale was different, and we found

$$a=218 \text{ centims.}$$

also

$$\alpha = \cdot 0003$$

$$\rho = \cdot 000072 \text{ B.A. unit}$$

α being the coefficient of increase of resistance of R per B.A. unit per degree, ρ the resistance of 1 millim. of the bridge.

The values of the resistances S and V were different from those in Part I., for the piece of copper wire which was included in the value of V in Part I. had been removed while the variable resistance T was no longer in circuit with S.

The values employed in the calculations are

$$V = 1{\cdot}00096 \text{ units.}$$
$$S = 3059{\cdot}89 \quad ,,$$

so that

$$\frac{V+S}{V} = \frac{3060{\cdot}891}{1{\cdot}00096}$$

The temperature of V and the 3000 ohms, which was in the same water bath with it, being 12°, that of the galvanometer 13°·5; this latter temperature being the mean of the temperatures of the galvanometer during the observations.

As before, we shall describe later the methods used to determine these values.

Table IV. gives the results of the calculations.

The mean value of R_0 deduced from it is

$$158{\cdot}386 \frac{\text{earth quadrant}}{\text{second}}$$

As we shall see afterwards, the value of R_0 in B.A. units is 160·520.

Thus

$$1 \text{ B.A. unit} = \frac{158{\cdot}386}{160{\cdot}520} \text{ ohms}$$
$$= {\cdot}986706 \text{ ohm.}$$

TABLE IV.

Series.	Battery.	M.	$\frac{11p^2-8q^2}{32a^2}$	R_0 in ohms.	Error in R_0 from mean of series.	Mean value of R_0 for series.	Error from mean.	Percentage error.
A	4 D.	$1{\cdot}55587 \times 10^8$	·00167	158·233	−·158	158·391	·005	·003
	4 D.		·00167	158·472	·081			
	5 TH.		·00263	158·296	−·095			
	2 D.		·00045	158·564	·173			
B	5 TH.	$1{\cdot}25758 \times 10^8$	·00136	158·171	−·099	158·270	−·116	−·072
	5 TH.		·00136	158·368	+·098			
C	5 TH.	${\cdot}762092 \times 10^8$	−·00019	158·397	−·061	158·458	·072	·045
	6 TH.		−·00015	158·301	−·157			
	6 TH.		−·00015	158·676	·218			

Mean of whole series 158·386 ohms.

Mean error for each series ·064.

Mean percentage error ·040.

Taking each series separately, the values we obtain are

Series A ·98673.
„ B ·98598.
„ C ·98716.

The greatest difference is ·00118, or about ·12 per cent.

Again, if we refer to the Table IV., we cannot find any clear indication of an error depending on the length of the rods used to separate the coils. The values of R_0 in series B are, it is true, somewhat small; there are, however, two values in series A and one in series C which are smaller than one of those in series B.

Neither do we find any connexions between the differences and the electromotive force used. Arranging the values in order of electromotive force, we have

2 DANIELL'S	158·564	1 experiment.
4 „	158·352	Mean of 2.
5 THOMSON'S	158·209	„ 4.
6 „	158·488	„ 2.

We must notice, however, that only one experiment was made with the smallest electromotive force.

It will be instructive to arrange the results in order of magnitude, noting the series, the battery, and the date of each experiment.

We have

Date.	R_0.	Series.	Battery.	Error.	Percentage error.
Dec. 2	158·171	B	5 THOMSON'S .	—·215	—·135
Nov. 16	158·233	A	4 DANIELL'S. .	—·153	—·095
„ 21	158·296	A	4 „ . .	—·090	—·056
„ 25	158·301	C	6 THOMSON'S .	—·085	—·053
Dec. 2	158·368	B	5 „ . .	—·018	—·011
Nov. 23	158·397	C	5 „ . .	·011	·007
„ 18	158·472	A	4 DANIELL'S. .	·086	·053
„ 16, 18, and 21 . .	158·564	A	2 „ . .	·178	·111
„ 28	158·676	C	6 THOMSON'S .	·290	·181

An inspection is sufficient to show that there is no definite order in any column but the second with the fifth and sixth, which are consequences of it. The greatest difference between any two experiments is ·505, and this in 158 is rather less than 1 in 300, or about ·32 per cent.

Of the actual errors of each experiment from the mean, five are negative and four are positive; the mean error itself is only ·125; the mean percentage error is ·078.

The number of experiments made is too small for the calculation of the probable error to have any value, but the distribution of errors round the mean is satisfactory, and the mean percentage error is very small, if we consider the complicated nature of the observations and the variation in the important conditions.

We turn now to the measurement of the resistances of the coil used in Part II. and their comparison with the B.A. standards.

It will be remembered that in the comparison between the coil Flat of the B.A. units and the 1 ohm of the box used in Part I., the correction to be made for the copper pieces connecting the box to the wire bridge and the resistance of the plugs was thought to introduce some error, which, however, it was shown would not affect seriously the value of the B.A. unit. To reduce this error the 1-unit coil of the box was not compared directly with the B.A. standard. Lord RAYLEIGH had had wound two 5-unit coils and one 10-unit, which had been carefully compared by him with the B.A. units. The comparison was repeated by one of us (R. T. G.), and the differences between the two results were found to be so small that we could use either value with all the accuracy required.

Thus the values at 12° were

5 units . . .	4·99392	Lord RAYLEIGH.
	4·99376	R. T. G.
10 units . . .	9·98360	Lord RAYLEIGH.
	9·98393	R. T. G.

The second 5-unit coil was only measured by Lord RAYLEIGH and Professor FLEMING; it belonged to the latter, and had been taken away by him before our comparison was made.

A third 5-unit, denoted afterwards by 5′, however, had been constructed for the laboratory to replace it, and its value was found to be 5·00890 at 12° C., while Professor FLEMING's coil had a resistance of 5·02444. Lord RAYLEIGH's value of the 10 units was found by comparison with the 5 units + FLEMING's 5 units in series; our value was obtained by comparison with 5+5′. The close agreement between the two results is sufficient test of the accuracy of the comparisons.

In determining the values of the resistances of the boxes, we started from these 5 and 10-unit coils. Two boxes were used—one by Messrs. ELLIOTT Brothers, No. 229, the other by WARDEN and MUIRHEAD, No. 202. The 10-unit coils in each of these boxes were compared with our 10-unit standard, using the modification of CAREY-FOSTER's method already employed to compare R and the resistance of the secondary circuit. Then 20 units in the box, made up in three different ways (viz.: by taking out (*a*) plug 20; (*b*) plugs 10 and 10′; (*c*) plugs 1, 2, 2, 5, and 10), was compared with the 10-unit and two 5-unit standards in series.

A large number of determinations were made both by R. T. G. and E. B. S. on different occasions. The various values obtained for the ELLIOTT box, reduced to a temperature of 12°, are given below; the temperature is that recorded by a thermometer laid on, or on some occasions inside, the box. Each number is the mean of three or four measurements taken at the same time.

Plug out.	Value.	Observer.
10	9·9903	R. T. G.
	9·9902	R. T. G.
	9·9890	E. B. S.
10′	9·9887	R. T. G.
	9·9907	R. T. G.
	9·9908	E. B. S.
1+2+2+5	9·9870	E. B. S.
20	19·9772	R. T. G.
	19·9775	E. B. S.
10+10′	19·9769	R. T. G.
1–10	19·9756	R. T. G.

For the WARDEN and MUIRHEAD box the differences between two sets of experiments were quantities of the same order as here.

The two boxes were then placed at opposite ends of the bridge-wire, the other two arms of the WHEATSTONE's bridge being the pair of 30-ohm coils used in the previous part of the experiments.

The 50-unit plug was taken out of one box, and out of the other all the plugs from 1–20. The difference between these two nominal 50 units was thus obtained. Then plugs 1–20 were taken out of the first box, and the 50 units out of the second, and another difference obtained. In this manner the values of the coils 50, 100, 100′, and 200 were obtained.

After this the differences between the two boxes became too large to be measured in terms of the resistance of the bridge-wire, and recourse was had to the method employed in Part I., by which one coil in a box was compared with a combination of coils in the same box.

Two coils, each of about 5 ohms, were connected with the ends of the bridge-wire, while the coils to be compared, P and Q, formed the other arms of the bridge.

Let $5+\alpha$, $5+\beta$ be the resistances of the two 5-ohm coils, and let x, y, x', y' have the usual meanings.

Then

$$\frac{P}{Q}=\frac{5+\alpha+x}{5+\beta+y}$$

Intercharge the 5-ohm coils

$$\frac{P}{Q}=\frac{5+\beta+x'}{5+\alpha+y'}$$

Hence

$$\frac{P}{Q}=1+\frac{\alpha-\beta+x-y}{5}$$

$$=1-\frac{\alpha-\beta-x'+y'}{5}$$

neglecting $\left(\frac{x}{5}\right)^2$ and such terms.

Now the values of the 5 units being known, we know α and β, and we find at the temperature of the observation

$$\frac{\alpha-\beta}{5}=\cdot0061$$

Hence

$$P-Q=Q\left\{\cdot0061+\frac{x-y}{5}\right\}$$

and

$$P-Q=Q\left\{\frac{x'-y'}{5}-\cdot0061\right\}$$

The values of x, y, &c., actually obtained were such as fully to justify the neglect of $\left(\frac{x}{5}\right)^2$ and such terms. By this means the values of the 500, 1000, 1000′, and 2000-unit coils in the ELLIOTT box were determined.

Having thus determined the values of the resistances of the box, they were used to determine that of R in the following manner.

R was connected with one end of the wire of the bridge, and the two boxes in multiple arc with the other, two 30-unit coils forming as before the third and fourth arms.

170 units were then taken out of the WARDEN box, and the ELLIOTT box adjusted until the difference between R and the total resistance of the compound circuit formed by the two boxes could be measured in terms of the bridge-wire. This was the case when 2920 units were out of the box. The actual value of these resistances at 12° is known from our table of resistances of the box. Making the correction for temperature and for the difference between R and the multiple arc resistance, we find

$$R_0=160\cdot602 \text{ B.A. units.}$$

A second determination, in which 180 units were out of one box and 1490 in the other, gave

$$R_0=160\cdot570 \text{ B.A. units.}$$

Each of these results is the mean of several experiments.

We may take, therefore, as the value of R_0 the mean

$$R_0=160\cdot586 \text{ B.A. units.}$$

The value of S, the resistance of the circuit through which the fractional part of the current ran when the deflection of the galvanometer was observed, was determined by direct comparison with the ELLIOTT box.

The values

$$3060 \cdot 71 \text{ B.A. units}$$

and

$$3060 \cdot 90 \text{ B.A. units}$$

were obtained as the means of several determinations on two different occasions.

The mean is

$$S = 3060 \cdot 80$$

The value of V was determined by comparison with the B.A. coil Flat, using the form of bridge devised by Professor FLEMING.

We found on four separate occasions the values

1·00096
1·00096
1·00095
1·00098

The temperature throughout at which these values are correct is 12°.

These methods involve the use of the resistance boxes, and, of course, there must be some uncertainty in the temperature of the coils in the interior of the box. Another series of determinations, therefore, was made of the values of R and S, in which the boxes were only used to determine the values of certain small corrections.

There is at the laboratory a coil of about 24 B.A. units—the standard used by Lord RAYLEIGH in his experiments on the value of the B.A. unit. This he had compared with the B.A. units and found that at 12° its value was 23·92820. He suggested that we should compare our standard of about 160 units directly with it, and this was done by one of us (R. T. G.).

I first determined *ab initio* the value of the 24 ohms in terms of the B.A. units. I found by a method to be afterwards described the three values

23·92807
23·92810

and

23·92850

The mean is

23·92822

which is practically identical with that found some months previously by Lord RAYLEIGH.

Now the ratio of 24 to 160 is between 1 : 6 and 1 : 7.

I therefore arranged a WHEATSTONE's bridge in the following manner:—

Two arms were the two standards I wished to compare; the coil R of about 160 units and Lord RAYLEIGH's standard, which we will call X, of about 24.

The third arm was two single unit coils and a 5-unit arranged in series, and the fourth arm a single unit, the coil Flat.

The two single units were the B.A. units C and G of Professor CHRYSTAL's report, their values and that of the 5-unit are accurately known in terms of the mean B.A. unit.

In all cases the electrodes of the coils were well amalgamated and rested securely on the copper discs at the bottoms of mercury cups.

The electrodes of the ELLIOTT box were connected with the two ends of the 7-unit arm, so that the box formed a shunt, and by altering the plugs in the box the effective resistance of the arm could be finely adjusted, and the ratio of the resistance of the Flat coil to that of this arm made equal to that of R to X.

Thus I found that with 164 units out of the box there was a deflection of the galvanometer of $+33{\cdot}2$ scale divisions, while with 163 out the deflection was $-5{\cdot}6$.

Thus the true value of the shunt is 163·143. Correcting this to the proper temperature we find that the effective resistance of this third arm is 6·70438 units, that of the Flat coil at the same temperature being ·99944 unit.

Now the temperature of R at the time of observation was 13°·3, that of X being 13°·4.

Hence

$$\frac{R_{13\cdot3}}{X_{13\cdot4}}=\frac{6\cdot70438}{\cdot99944}$$

Whence substituting the value of X and reducing to the standard temperature 12°

$$R_0=160{\cdot}523 \text{ B.A. units.}$$

A second experiment was made at a different temperature, and instead of noting the deflections of the galvanometer produced by altering the box by 1 unit a second shunt was introduced and varied until the deflection was zero: the value of this shunt was 30,000 units.

From this experiment I found

$$R_0=160{\cdot}518 \text{ B.A. units.}$$

We take as the true value of R_0 deduced from these two experiments

$$R_0=160{\cdot}520 \text{ B.A. units.}$$

We have now to compare directly the values of S_0 and R_0. S_0 is about 3060 units, so that the ratio of S_0 to R_0 is between 19 to 1 and 20 to 1.

Four sets of coils were, therefore, arranged for a WHEATSTONE's bridge, two arms of which were S and R, the other two arms being 20 units—made up of the two 5 and 10-unit standards already described, while the fourth was the single coil G.

The box was used as a shunt to the 20-unit arm, and adjusted as before till the galvanometer showed no deflection, the effective resistance of this arm was found to be 19·0444 units, while at the same temperature the value of G is ·99887 unit.

Substituting the value of R at the temperature of the observation and reducing our result to the temperature 12°, we find

$$S_0=3059{\cdot}37 \text{ units}$$

A second series of observations on a different occasion gave

$$S_0=3059{\cdot}86 \text{ B.A. units}$$

The mean is

$$S_0=3059{\cdot}62 \text{ B.A. units}$$

and this is the value we have used in our calculations.*

The coils were in all cases placed in water baths and allowed to stay for some hours in them. Before making the observations the temperature was read by thermometers graduated to fifths of a degree, which were compared with each other.

To determine then the value of the B.A. unit we must use the value of $R_0=160{\cdot}520$, found in this series of measurements. Now the values of S_0 and R_0 found from the boxes were respectively

$$3060{\cdot}80 \text{ and } 160{\cdot}586$$

These differ from the values we have obtained in our last observations by 1·18 and ·066 unit respectively, or rather more than 1 in 3000. This difference would correspond to an error of about 1° in the measurement of the temperature.

Considering then the uncertainty which must attach to the temperature of the coils inside the box, it seemed best to take our last values rather than the mean of the two —we would rather regard the first series as a check upon any large error. But though this difference in the value of S will affect to the amount of 1 in 3000 the value of R_0 in absolute units, it does not affect at all sensibly the value of the B.A. unit, for this latter depends on the ratio of S_0/R_0; taking the values of S_0 and R_0 from the boxes, we have

$$\frac{S_0}{R_0}=19{\cdot}0602$$

while, if we use the last values obtained for S_0 and R_0, we get

$$\frac{S_0}{R_0}=19{\cdot}0607$$

the difference is only about 1 in 40,000 and does not concern us. We shall therefore put $R_0=160{\cdot}520$ ohms, the temperature being 12°.

* The value 3059·89 used on page 255 is obtained from this by applying a temperature correction to the 60 units, the resistance of the galvanometer, which was at 13°·5.

The contrivance designed by Lord RAYLEIGH by means of which our 24-unit standard was compared with the B.A. units needs a special reference: it has been described by Lord RAYLEIGH in his second paper on the value of the B.A. unit, an abstract of which was read before the Royal Society on March 9, 1882, while the paper is published in the Phil. Trans., Part I., 1882.

Five coils each of approximately 5 ohms resistance were wound and enclosed in a box, from which the two electrodes, copper rods with amalgamated ends, of each coil protrude. By means of two series of mercury cups this system could be put either in series or in multiple arc. Then, if each coil of the series is so nearly equal to 5 units that we may neglect the square of the difference, it is easy to show that the resistance of the system in series is exactly 25 times that which it has when in multiple arc. The coil of 24 units and a single unit were arranged in series so that they could readily be put into connexion with FLEMING'S bridge. The set of five 5-unit coils in multiple arc was compared with a single unit. The connexions were rapidly altered, and the five 5 units in series were compared with the 24+1; then again adjusting the connexions, another comparison between the 25 units in multiple arc and the single was made.

In this manner a value of the 24 ohms was obtained in terms of the single ohm, the result of the comparison being as already stated.

Thus, whether we use as the values of R_0 and S_0 those found from comparison with the box or those determined by comparison with the coils, we have as values of the B.A. unit determined from this second part

Series A	·98673	mean of	4
,, B	·98598	,,	2
,, C	·98716	,,	3

While the mean result is

·986706 ohm.

The result obtained in June, 1881, Part I., as the mean of three complete sets, was

·986350 ohm.

Our discussion has shown us that the possible errors of this determination are considerably greater than that obtained in Part II. We will, therefore, give to each experiment in Part I. only half the weight of an experiment in Part II., and obtain thus as our final value for the ohm

·98665 ohm.

The value obtained by Lord RAYLEIGH in his second experiments with the rotating coil (Phil. Trans., Part I., 1882) is

·98651* ohm.

* Since this paper was read Lord RAYLEIGH has obtained by two modifications of LORENZ' method the two values ·9867 and ·9868.

The difference, amounting as it does to about 14 parts in 100,000, is obviously less than the probable error of our result, and there can be little doubt but that the value ·9866 is only two or three parts in 10,000 from the truth. This value, as Lord RAYLEIGH has pointed out, is strikingly confirmed by JOULE's latest determinations of the mechanical equivalent of heat.

In conclusion, we would discuss further one or two possible sources of error. We have assumed that the ratio of V/S is independent of the temperature. This is not true, for S consists of two parts; one part of about 3000 units has the same temperature coefficient and is always at the same temperature as V, so that for it our assumption is justified; the other part of the galvanometer, of about 60 units, is of copper, of which the temperature coefficient is ·003, and it is not at the same temperature as V.

Now the whole range in the temperature of the galvanometer is about 3°, the range on either side of the mean 1°·5, and an error of 1°·5 in the temperature of the galvanometer we can show would affect R to the amount of about one part in 8000. Thus those values of R for which the temperature of the galvanometer differs most from the mean may be affected to the amount of 1 in 8000 by this source of error. Taking the mean of all our observations, however, the error vanishes.

Professor CHRYSTAL had warned us that he had had some difficulty in securing sufficiently good insulation between the wire of the coils A and B and the brass rings in which they were wound. We therefore tested both coils to see that this was maintained. One pole of a battery of 25 LECLANCHÉ cells was connected with the brass ring, while the other pole was put in contact with one electrode of a galvanometer of 2000 ohms resistance, the second electrode of the galvanometer being connected with the wire of the coil.

Deflections of 60 and 80 divisions of the scale were observed for the two coils A and B respectively. The same battery through 100,000 ohms when the galvanometer is shunted with 8 ohms gave a deflection of over 100 divisions, the insulation resistance therefore is considerably over

$$\frac{100,000 \times 2000}{8}$$

or

$$25,000,000 \text{ ohms.}$$

The error that might arise from the use of a paper scale has been discussed. In the calculations corrections have been applied to the scale readings to reduce them to the standard metre. The corrections taken were the mean of four series of observations—two by R. T. G., two by E. B. S.; and these, though made at very different times, varied only by quantities comparable with the error of an observation.

The most serious objection, however, that can be raised applies to all observations in which a ballistic galvanometer is used, and there were two points here which seemed to require special notice. Is it right to assume that on reversing the primary current the change that takes place is over in a time which is small compared with the time of swing?

Now our reversals were made with an ordinary rocking commutator dipping into mercury cups, and only occupied a small fraction of a second. We therefore made a series of experiments to see if we could find the effect produced by holding the primary circuit open for definite small periods.

On reversing in the usual rapid manner we obtained a throw which varied between 119·5 and 119·3 millims.

When our primary circuit was held open for one second the throws were 118·9, 119·3, 119·3, and 119·1, while if the contact was broken for two seconds the effect on the throw was marked. Thus if by chance in any experiment the primary circuit was open for as much as a second, so that the battery current took something over a second to get steady, it would only produce an effect of about 1 in 1000 in the result; we feel quite certain that in no case the period of break occupied more than a small fraction, from a quarter to one-sixth, of a second.

Again, it might happen that the somewhat powerful induced current passing through the galvanometer coils might alter—temporarily or permanently—the magnetic moment of our needle.

A permanent alteration would of course be indicated by variations both in the throw produced by the induction current and in the time of swing. In the actual experiments after the induction current had been passed in one direction through the coils of the galvanometer, and the throw observed, it was passed in the other direction to stop the vibrations of the needle, and it was possible that each current might have produced real permanent changes in the magnetic moment, but of exactly equal amount, so that we had observed no appreciable alterations in the time of swing which we could assign to this cause.

We therefore took a series of measurements of throws in which the current was only allowed to pass in one direction through the coils, the vibrations of the needle being stopped by means of an external damper; the times of swing also were observed at intervals.

The table below gives the result.

Time of swing.	Throw.
	115·8
23·383	
	116·0
	116·0
	115·7
	116·0
	115·9
23·389	

SECOND series.—Throws in opposite direction.

Time of swing.	Throw.
	115·5
23·389	
	115·6
	116·2
	115·8
23·392	

The throws throughout are very nearly equal; the time of swing, it is true, has increased by ·009″, but this change cannot be due to an alteration in the magnetic moment of the needle, for if the current passed in the one direction had decreased the magnetic moment and so increased the time of swing, that passed in the other direction in the second series would have increased the magnetic moment and so decreased the time of swing. The alteration in T then we must rather ascribe to a variation in the value of the horizontal intensity. This has been allowed for in the experiments by the repeated observations which were made of T. As regards the temporary effect of the induced magnetism produced by the current while it lasts, we notice that its direction would at each instant be perpendicular to the plane of the coils, so that the force between the magnet thus formed by the current and the current forming it would be along the axis of the temporary magnet and have no tendency to cause it to move. In fact, the field of force produced by the coils being uniform, the actual force on the induced magnetism will be zero.

A number of observations were also made to see if the time taken by the battery current to become steady after a reversal was appreciable. The galvanometer circuit being open the battery circuit was reversed, and then the galvanometer circuit closed at an interval of from ·25″ to ·3″ after.

If after this interval the battery current had still been varying an induction current would have shown itself in the galvanometer, but though the reversals were made in three separate experiments, 13, 7, and 10 times respectively, in time with the period of the galvanometer, no effect was produced.

We are sure, therefore, that the battery current has attained its steady value in less than ·3″ after reversal, while our former experiments have shown us that if the period of change were as great as 1″ the error produced in the throw would be less than 1 in 1000.

The direct effects of the connecting wires and commutators on the galvanometer during the experiments were carefully tested for but no result could be found. Copper wire insulated with gutta-percha was used for the connexions, and the wires in each circuit were carefully twisted up together.

Most of the apparatus employed belongs to the Cavendish Laboratory, where the experiments were conducted. Our best thanks are due to Lord RAYLEIGH for his unceasing kindness and his many valuable suggestions which have helped us satisfactorily to surmount several serious difficulties, and have added greatly to the accuracy of the results.

NOTE.

(Added December 2, 1882.)

Another source of error should have been mentioned in the previous discussion.

The external diameter of each layer was measured necessarily before the layers external to it were wound on. There will probably be a tendency in each layer to compress those below it, and thus to make the mean radius of the coil somewhat less than we have assumed.

In fact, Lord RAYLEIGH found in his experiments (Phil. Trans., Part I., 1882) that the value of the mean radius obtained from measurements made as the coil was unwound was less by 1 in 2000 than that obtained from the measurements made while winding.

Several reasons lead us to believe that any effect of the kind would be much less with our coils.

The error is produced probably by the silk being pressed over the top and bottom of the layer into the intervals between the wires—the wires being circular in section there is necessarily a space between them.

Now the amount of this free space would be proportional to the area of the cross section of the wire, while the amount of silk would depend on the circumference of this cross section.

The diameter of our wire was only about two-thirds of that used by Lord RAYLEIGH, there would, therefore, be less space in proportion to the amount of silk into which the silk could be squeezed, and the error produced would be less.

Again, if this yielding is going on, it is clear that the average thickness of a layer should appear to decrease as we get near the outside surface. A reference to the measurements shows that this is not the case. On dividing the whole series of layers into sets of five, and taking the average for each set, omitting a layer in which paraffin paper had been used, we find the values for the average thickness ·128, ·123, ·127, ·128, and ·127.

No gradual decrease is here observable. We therefore feel confident that the error was much less than in the coils wound by Lord RAYLEIGH.

A strong indirect confirmation of the accuracy of the value of the mean radius used is afforded by Lord RAYLEIGH'S recent measurements by LORENZ' method.

The same coils were used as in our observations, and Lord RAYLEIGH has shown that an error in the mean radius would affect the two methods by about the same amount but in opposite directions. If δM be the error in M produced by an error δa in a the mean radius, it follows that in our case $\frac{\delta M}{M}$ is about $2\frac{\delta a}{a}$, while in LORENZ' method, M being the induction between the disc and the coils, for the arrangement adopted $\frac{\delta M}{M} = -2\frac{\delta a}{a}$ approximately—in the one case δM is of the same sign as δa, in the other the signs are opposite. The mean of the results then obtained from the two methods will be free from an error in a. But the two results are ·98665 and ·9867. They are identical to the fourth figure. Thus we infer that the error in a is probably very small.